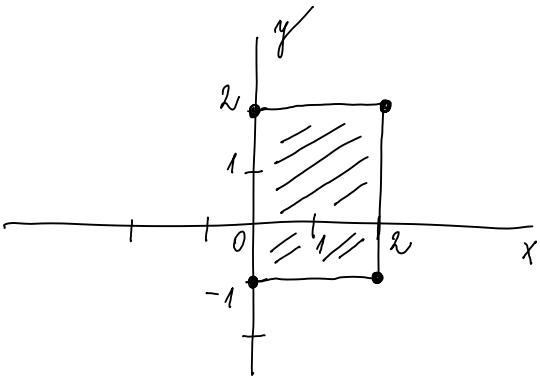
Úloha 5 - použito 31. 5.

čtvrtek 4. června 2020 11:50

$g(x,y) = x^3 + y^3 - 3xy$

obdélník o vrcholy $[0,-1], [2,-1], [2,2], [0,2]$



1) extrémy uvnitř □

$\frac{\partial g}{\partial x} = 3x^2 - 3y$

$\frac{\partial g}{\partial y} = 3y^2 - 3x$

$3x^2 - 3y = 0 \;/:3$

$3y^2 - 3x = 0 \;/:3$

$x^2 - y = 0 \rightarrow y = x^2$

$y^2 - x = 0$

$(x^2)^2 - x = 0$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x_1 = 0 \quad y_1 = 0^2 = 0 \rightarrow A[0,0]$ .. není uvnitř □

$x_2 = 1 \quad y_2 = 1^2 = 1 \rightarrow B[1,1]$

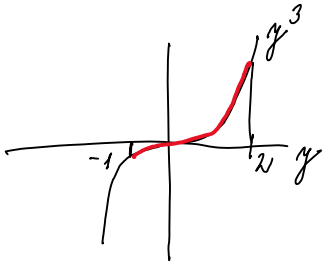
$\frac{\partial^2 f}{\partial x^2} = 6x \qquad \frac{\partial^2 f}{\partial x \partial y} = -3$

$\frac{\partial^2 f}{\partial y^2} = 6y \qquad \frac{\partial^2 f}{\partial y \partial x} = -3$

$D(B) = \begin{vmatrix} 6 & -3 \\ -3 & 6 \end{vmatrix} = 36 - 9 = 27 > 0$

→ je extrém (LMIN)

2) extrémy na hranici □



a) $x = 0, \; y \in \langle -1, 2 \rangle$

$g(y) = y^3$

$y = -1$ .. LMIN → $C[0,-1]$

$y = 2$ .. LMAX → $D[0,2]$

b) $x = 2, \; y \in \langle -1, 2 \rangle$

$g(y) = 8 + y^3 - 6y = y^3 - 24y + 8$

$g(y) = 8 + y^3 - 6y = y^3 - 24y + 8$

$g'(y) = 3y^2 - 6$

$3y^2 - 6 = 0$

$y^2 = 2$

$y_1 = \sqrt{2}$

$y_2 = -\sqrt{2}$ ... neleží v $\langle -1, 2 \rangle$

$y = -1$ .. LMAX → $E[2, -1]$

$y = 2$ ... LMIN → $F[2, 2]$

$(-\infty, -\sqrt{2})(-\sqrt{2}, \sqrt{2})(\sqrt{2}, +\infty)$

$g'(y)$ + − +

↗ ↘ ↗

LMIN

$H[2, \sqrt{2}]$

c) $y = -1$, $x \in \langle 0, 2 \rangle$

$g(x) = x^3 - 1 + 3x = x^3 + 3x - 1$

$g'(x) = 3x^2 + 3$

$3x^2 + 3 = 0$ NŘ

$x = 0$ ... LMIN → $C[0, -1]$

$x = 2$ ... LMAX → $E[2, -1]$

d) $y = 2$, $x \in \langle 0, 2 \rangle$

$g(x) = x^3 + 8 - 6x = x^3 - 6x + 8$

$g'(x) = 3x^2 - 6$

$3x^2 - 6 = 0$

$x^2 = 2$

$x_1 = \sqrt{2}$ ... LMIN → $G[\sqrt{2}, 2]$

$x_2 = -\sqrt{2}$ ... neleží v $\langle 0, 2 \rangle$

$(-\infty, -\sqrt{2})(-\sqrt{2}, \sqrt{2})(\sqrt{2}, +\infty)$

$g'(x)$ + − +

↗ ↘ ↗

LMIN

$x = 0$ .. LMAX → $D[0, 2]$

$x = 2$ .. LMAX → $F[2, 2]$

3) porovnání funkčních hodnot

$B[1, 1]$ $f(B) = -1$ } GMIN

$C[0, -1]$ $f(C) = -1$

$D[0, 2]$ $f(D) = 8$ GMAX

C [0;-1]

D [0;2]

E [2;-1]

F [2;2]

G [√2;2]

H [2;√2]

$f(D) = 8$

$f(E) = 13$ GMAX

$f(F) = 4$

$f(G) = 8 - 4\sqrt{2} \doteq 2{,}3$

$f(H) = 8 - 4\sqrt{2} \doteq 2{,}3$