Test 1 - úloha 1

pátek 5. března 2021 20:05

Najděte řešení rovnice $\sqrt{y}\cos x = y'\sin^2 x$ procházející bodem $[\frac{\pi}{2}, \frac{1}{4}]$

$$\sqrt{y}\cdot\cos x = y'\sin^2 x$$

$$\sqrt{y}\cos x = \frac{dy}{dx}\sin^2 x$$

$$\int \frac{dy}{\sqrt{y}} = \int \frac{\cos x}{\sin^2 x}\,dx$$

$$\int y^{-\frac{1}{2}}dy = \int \frac{\cos x}{\sin^2 x}\,dx$$

$$2\sqrt{y} = -\frac{1}{\sin x} + C \quad /:2$$

$$\sqrt{y} = -\frac{1}{2\sin x} + C \qquad C - \frac{1}{2\sin x} \geq 0$$

$$y = \left(C - \frac{1}{2\sin x}\right)^2$$

$$y = 0$$

$$\int \frac{\cos x}{\sin^2 x}\,dx = \left| \begin{array}{l} u = \sin x \\ \frac{du}{dx} = \cos x \\ du = \cos x\,dx \end{array} \right| = \int \frac{1}{u^2}\,du = \int u^{-2}du = -\frac{1}{u} = -\frac{1}{\sin x}$$

$$\frac{1}{4} = \left(C - \frac{1}{2\sin\frac{\pi}{2}}\right)^2$$

$$\frac{1}{2} = C - \frac{1}{2} \qquad -\frac{1}{2} = C - \frac{1}{2}$$

$$C = 1 \qquad C = 0$$

$$y_p = \left(1 - \frac{1}{2\sin x}\right)^2 \qquad y_p = \frac{1}{4\sin^2 x}$$

Test 1 - úloha 2

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najděte obecné řešení rovnice $y' = \frac{2y}{x+1} + (x+1)^3$

$$y' - \frac{2y}{x+1} = (x+1)^3$$

1) $y' - \frac{2y}{x+1} = 0$

$$y' = \frac{2y}{x+1} \qquad y \neq 0$$

$$\int \frac{1}{y}\,dy = 2\int \frac{1}{x+1}\,dx$$

$$\ln|y| = 2\ln|x+1| + C$$

$$|y| = e^{2\ln|x+1|+C}$$

$$|y| = e^{\ln(x+1)^2} \cdot e^C$$

$$|y| = C(x+1)^2 \quad C \in R^+$$

$$y = C(x+1)^2 \quad C \in R$$

2) $y_0 = C(x)(x+1)^2$

$$y' = C'(x)(x+1)^2 + C(x)\,2(x+1)$$

$$C'(x)(x+1)^2 + 2\cancel{C(x)(x+1)} - \frac{2\cancel{C(x)(x+1)}^2}{x+1} = (x+1)^3$$

$$C'(x)(x+1)^2 = (x+1)^3$$

$$C'(x) = x+1$$

$$C(x) = \int (x+1)\,dx$$

$$C(x) = \frac{x^2}{2} + x + K$$

$$y_0 = C(x)(x+1)^2 = \left(\frac{x^2}{2} + x + K\right)(x+1)^2 = K(x+1)^2 + \left(\frac{x^2}{2} + x\right)(x+1)^2$$

Test 1 - úloha 3

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najděte obecné řešení rovnice $2y' \cdot y = x$

$$2y' \cdot y = x$$

$$y' y = \frac{x}{2}$$

$$\int y\,dy = \frac{1}{2}\int x\,dx$$

$$\frac{y^2}{2} = \frac{1}{2}\left(\frac{x^2}{2} + C\right) \quad / \cdot 2$$

$$y^2 = \frac{x^2}{2} + C$$

$$y = \pm\sqrt{\frac{x^2}{2} + C}$$