

Test 2 - úloha 1

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20:52

$$y'' - y' - 2y = e^{3x}$$

1) homogenní DK

$$y'' - y' - 2y = 0$$

$$\alpha^2 - \alpha - 2 = 0$$

$$\alpha_1 = -1$$

$$\alpha_2 = 2$$

$$y_0 = C_1 e^{-x} + C_2 e^{2x}$$

2) nehomogenní DK

$$f(x) = P(x)e^{cx} \cos dx + Q(x)e^{cx} \sin dx = e^{3x}$$

$$\left. \begin{array}{l} c = 3 \\ d = 0 \end{array} \right\} c + di = 3 \rightarrow \kappa = 0$$

$$\left. \begin{array}{l} P(x) = 1 \\ Q(x) = 0 \end{array} \right\} P_1(x) = a$$

$$y_N = x^\kappa (P_1(x)e^{cx} \cos dx + Q_1(x)e^{cx} \sin dx)$$

$$y_N = x^\kappa P_1(x) e^{3x}$$

$$y_N = a e^{3x}$$

$$y'_N = 3a e^{3x}$$

$$y''_N = 9a e^{3x}$$

$$9a e^{3x} - 3a e^{3x} - 2a e^{3x} = e^{3x}$$

$$4a e^{3x} = e^{3x}$$

$$4a = 1$$

$$a = \frac{1}{4}$$

$$y_N = \frac{1}{4} e^{3x}$$

$$3) y = y_0 + y_N = C_1 e^{-x} + C_2 e^{2x} + \frac{1}{4} e^{3x}$$

Test 2 - úloha 2

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$$y''' + y'' + y' + y = x^3$$

1) homogenní DK

$$y''' + y'' + y' + y = 0$$

$$\alpha^3 + \alpha^2 + \alpha + 1 = 0$$

$$\alpha^2(\alpha+1) + (\alpha+1) = 0$$

$$(\alpha^2+1)(\alpha+1) = 0$$

$$\alpha_1 = -1$$

$$\alpha_2 = i$$

$$\alpha_3 = -i$$

$$y_0 = c_1 e^{-x} + c_2 \cos x + c_3 \sin x$$

2) nelogenní DK

$$f(x) = P(x)e^{cx} \cos dx + Q(x)e^{cx} \sin dx = x^3$$

$$\left. \begin{array}{l} a = 0 \\ d = 0 \end{array} \right\} c + di = 0 \Rightarrow k = 0$$

$$\left. \begin{array}{l} P(x) = x^3 \\ Q(x) = 0 \end{array} \right\} P_1(x) = ax^3 + bx^2 + cx + d$$

$$y_N = x^k (P_1(x)e^{cx} \cos dx + Q_1(x)e^{cx} \sin dx)$$

$$y_N = ax^3 + bx^2 + cx + d$$

$$y'_N = 3ax^2 + 2bx + c$$

$$y''_N = 6ax + 2b$$

$$y'''_N = 6a$$

$$6a + (6ax + 2b) + (3ax^2 + 2bx + c) + (ax^3 + bx^2 + cx + d) = x^3$$

$$a = 1$$

$$3a + b = 0 \rightarrow b = -3$$

$$6a + 2b + c = 0 \rightarrow c = 0$$

$$6a + 2b + c + d = 0 \rightarrow d = 0$$

$$y_N = x^3 - 3x^2$$

$$3) y = y_0 + y_N = c_1 e^{-x} + c_2 \cos x + c_3 \sin x + x^3 - 3x^2$$

Test 2 - úloha 3

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$$\left. \begin{array}{l} y(0) = 28\,734 \\ y(5) = 121\,447 \\ y(8) = ? \end{array} \right\} \text{exponenciální růst}$$

$$y' = ky$$

$$\frac{dy}{dx} = ky$$

$$\int \frac{dy}{y} = k \int dx$$

$$\ln|y| = kx + C$$

$$|y| = e^{kx} \cdot e^C$$

$$y = Ce^{kx}$$

$$y = C(e^k)^x = Ca^x$$

$$y(0) = 28\,734$$

$$28\,734 = Ca^0$$

$$28\,734 = C$$

$$y(5) = 121\,447$$

$$121\,447 = 28\,734 a^5$$

$$a = \sqrt[5]{\frac{121\,447}{28\,734}}$$

$$a \doteq 1,33413$$

$$y = 28\,734 \cdot 1,33413^x$$

$$y(8) = 28\,734 \cdot 1,33413^8 = 288\,390 \text{ malakénový ořech}$$