Possible confrontations with cognitive conflict as a teaching strategy of mathematical concepts.

Magdalena Krátká, Jiří Přibyl

University of J. E. Purkyně in Ústí nad Labem, magdalena.kratka@ujep.cz, jiri.pribyl@ujep.cz

Abstract:

As one of the strategies for developing an understanding of mathematical concepts, teachers use (across grades) the cognitive conflict strategy. They present students with tasks or questions that they expect to evoke first contradiction and then "better" knowledge as a result of confronting an individual with cognitive conflict. In this article, we deal with four case studies where teachers wanted to provoke cognitive conflict in different contexts - in three cases, it is students between Grade 4 to Grade 6; in one case, it is a pre-service teacher. We try to show that the mere presence of contradiction does not necessarily mean the induction of cognitive conflict. Moreover, if the cognitive conflict is evoked, it does not necessarily mean that the student can find its core or that it solves it in the desired way.

Key words: cognitive conflict, mathematical concept, contradiction

Introduction

Our research shows one of the possibilities of using cognitive conflict (CoCo) as a tool for building mathematical concepts for pupils aged 9 to 12 (Grade 4 till Grade 6). CoCo may be perceived as one of the basic strategies of critical thinking from a broader perspective, with perceiving critical thinking as a necessary skill for the 21st century (Suh & Seshaiyer 2013). The basic idea of critical thinking falls to the past of European education, and this idea is based on Socratic dialogue. As a necessary part of the Socratic dialogue, we must perceive the (cognitive) conflict to which one of the actors is led by appropriately asked questions. (Gazda, Liška, & Marek, 2019) The settlement of this conflict moves the actor of dialogue (and through him, the reader) further.

Theoretical background

Piaget explains the process of learning based on an adaptation of the organism to the environment by means of the so called assimilation, when new knowledge falls in the knowledge structure of an individual, so that the cognitive structure can be just extended, and accommodation, when for the achievement of any new knowledge it is necessary to rebuild the existing knowledge structures. The process of learning is thus characterized by the constant disturbance of the balance between assimilation and accommodation. When the balance is upset, then the new piece of knowledge cannot be included in the existing cognitive scheme. In that case Piaget speaks about the cognitive conflict, that is about the psychological tension and the subsequent effort to solve the problem. The development of intelligence and reasoning is carried out in stages, always based on a certain new situation, which can turn over the current order and thus create the disequilibrium which will have to be compensated by an individual by means of adaptation from which a new, much stronger equilibrium will be created (Piaget, 2001).

Piaget's theory is then followed by other theorists and researchers who point out that the construction of mathematical concepts cannot be described only by positive stages. This construction is also characterized by overcoming obstacles, which appear to be a negative stage at the moment. The overcoming of obstacles, as mentioned above, is one of the usual activities that are regularly repeated in particular cycles and is an inseparable part of some phases of learning mathematical concepts. As one of the initial theories following Piaget's work, we chose the APOS theory (Action, Process, Object, Schema). From our perspective, we believe this theory appropriately describes the individual phases of the cognitive process. (Arnon et al., 2014)

Tall and Schwarzenberger speak about the so called conscious and subconscious conflicts induced by two "close" (mathematical) mutually irreconcilable concepts. They say that conflicts are induced by the transfer of mathematical ideas into the teaching and learning process when these are inevitably deformed. In a greater detail: in some cases, "the cause of the conflict can be seen to arise from purely linguistic infelicity...", in another case "the cause of the conflict arises from a genuine mathematical distinction.", and finally "the conflict arises from particular events in the past experience of an individual pupil." (Tall & Schwarzenberger, 1978, p. 49). Thus, we could say that the causes of CoCos can have both a didactic and an epistemological basis. It is worth noting that the ontogenetic nature of the conflict is not right because there is probably no conflict at the time, but only an external discrepancy.

We believe there are two main approaches to handling CoCo situations: direct assimilation, which involved fitting new information with what was already known, and knowledge building, which involved treating new information as something problematic that needed to be explained. (Limón, 2001). Chan, Burtis and Bereiter (1997) elaborated a knowledge-processing activity scale to evaluate individuals' reactions to the processing of contradictory information. It consisted of the following five levels:

- subassimilation when new information is reacted to at an associative level;

- direct assimilation when new information is assimilated either as if it was already known or excluded if it does not fit with prior beliefs. New information can be ignored, denied, excluded or distorted to make it fit with prior beliefs. Ad hoc rationalisations are also possible;
- surface-constructive when new information is comprehended, but its implications for one's beliefs are not considered. There is no integration of naive ideas with the new information. A new idea can be considered an exceptional case that does not involve the review of one's own beliefs or ideas;
- implicit knowledge building when new information is treated as something problematic that needs to be explained. Conflict is identified and new information is considered to be something different from one's beliefs. Inconsistencies are identified and explanations are built to reconcile knowledge conflict;
- explicit knowledge building when new information is accumulated for constructing coherence in domain understanding. Connections among the new information are sought and conflicting hypotheses are identified to explain the domain in question. (Chan, Burtis & Bereiter, 1997, p. 12))

Tall a Schwarzenberger (1978) further state, that if the conflict is conscious, we can expect the following reaction: "... the existence of two 'nearby' concepts can cause mental stress arising from the emergence of unstable thoughts" (p. 44). An individual makes an effort to remove this tension, which may or may not be a successful strategy for developing an understanding of a phenomenon or a concept.

Definition of cognitive conflict for the purpose of the experiment

CoCo is when the individual, based on their knowledge (concept, algorithm, scheme, ...), enters into a contradiction with some of their other knowledge, experience or current evidence, and at the same time becomes aware of this contradiction. So, they get into the imbalance (disequlibrium), the inner tension that Piaget was talking about. The realization of the evidence or the other knowledge can be achieved by the individual themselves, or this is presented to them by someone else, most often a teacher or classmate. However, CoCo does not have to occur during the presentation of the contradiction itself because the necessary condition is the presence of the inner psychological tension that elicits an appropriate response in the individual.

CoCo can be induced works with two conflicting mathematical concepts. One of them is already necessarily in the cognitive structure of the individual. The other one may be there also or may be implemented from outside, most often by a teacher. If the CoCo is induced, the individual is forced to evaluate which of the concepts changes or completely rejects. However, it is also possible that they perceive CoCo, but cannot reveal what concepts it is based on.

It depends mainly on the cognitive level / capacity of the individual and also on their mental maturity, whether any evidence or argument can cause CoCo. Given that, in our research, we are dealing with children aged 9 to 12, this is an essential circumstance because this age is a period of maturation of higher brain functions. Thus, the teacher can completely miss the pupil's cognitive ability if the core of the contradiction settles outside the pupil's cognitive maturity.

In our research, we rely on the appropriate inclusion of such situations that lead to CoCo in teaching. The idea of intentional induction of these situations as an educational tool is described in (Adnyani, 2020). The individual steps of the process leading to CoCo can be described as follows:

1. identifying students' current state of knowledge;
2. confronting students with unexpected information, ‘counterexample’ or a problem, that has the potential to cause CoCo. (Adnyani, 2020; Chan, Burtis, & Berereiter, 1997)

However, these are only the starting point of our research. We focused not only on the study of a pupils' reactions but also on pre-service teachers and their reactions.

Methodology

In the research, we focused on two groups of respondents. The first group consists of pupils, where we observed and recorded the courses of teaching situations. Teachers and researchers prepared these situations to cause the existence of CoCo with pupils. The second group of respondents consists of pre-service teachers (first year of the bachelor study program), which we exposed to CoCo to monitor their reactions and then analyse their responses. We believe this encounter with CoCo is an appropriate start point for discussing teaching strategies in a constructivist manner.

The teacher and researcher planned experimental teaching (creating specific teaching situations) for pupils involved in our research (age between 9 and 12 years). In all cases, experimental teaching was led by teachers with at least three years of experience. The prepared teaching situations fell into various mathematical contexts, and we did not limit their choice in any way.

In cooperation with the teacher, we determined the direction of teaching: what knowledge the planned CoCo should relate to, how the teacher should diagnose the presence of CoCo, and finally, what confrontation the teacher assumes. To this end, we asked the following questions to teachers:

(1) What mathematical concept does the didactic situation focus on, or what is it problematic in?

(2) What CoCo should occur? Describe a concept that should provoke CoCo.

(3) What are your expectations regarding the student's confrontation with this CoCo?

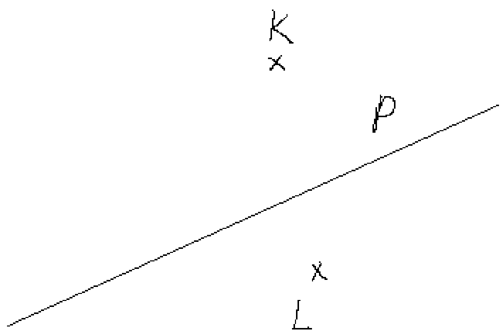
We recorded all teaching situations in writing and then performed a repeated qualitative analysis. First, we sought confirmation of the presence of CoCo, followed whether respondents declared an understanding of its nature, and finally observed whether and how the individual was trying to resolve CoCo. As part of this analysis, we identified the categories into which we classified the relevant passages of records of learning situations to describe the qualitative aspects of the strategy of CoCo.

Results

In this section, we will present several recordings of teaching situations. For each, we briefly state the teacher's assumptions and describe the key phenomena of the analysed situation.

<u>Case study No. 1: Jan and Dan, Grade 5; Teacher A</u>

Task: Is it possible to connect points K and L with line so that it does not intersect straight line p?



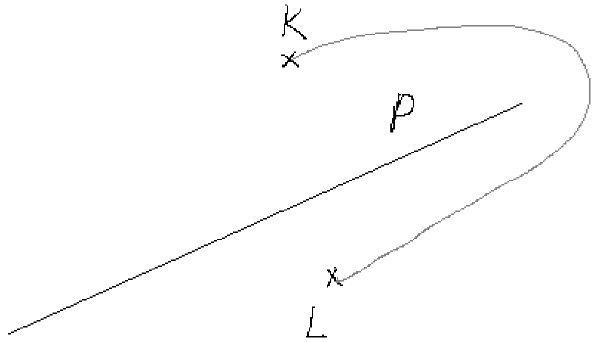
Teacher expectation:

(1) a straight line; it is understood as a drown line, not as a boundedness line

(2) the existence of intersection of two non-parallel lines

(3) pupils will be able to correct their statement after an additional task

Jan 1: Yes, it would be possible. [Jan connects the points K and L with a line avoiding the picture of the straight line p.]



Dan 2: You are right, it is also possible to go on the other side. [Dan indicates another line KL.]

Teacher 3: Hm [pause] So, the straight line p does not intersect the line KL, doesn't it?

Jan 4: Well [he nods], it does not.

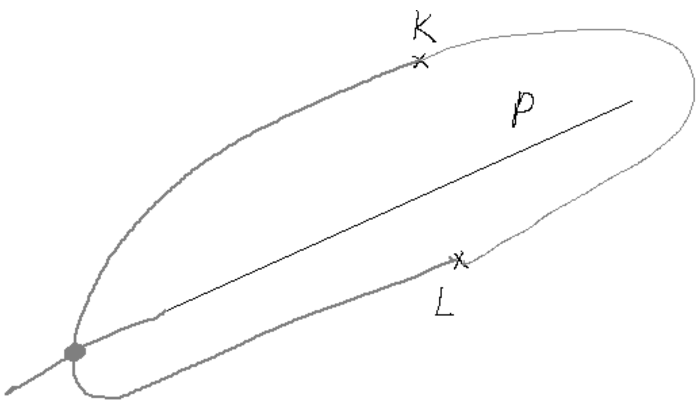
Teacher 7: OK, Look, there are two lines, p and q. Will they intersect?

Jan 8: Yes. [pause] We would find their intersection if we extend them. [He extends both lines and finds a point of intersection.]

The teacher persuaded the students to make a statement that he/she considered controversial with their original statement. Thus, he/she assumes that CoCo can be induced. The other question is to confirm this.

Teacher 9: Well done. So, would it be possible to find an intersection for your lines KL and for the straight line as well? [Teacher points to the original task.]

Dan 10: Sure, I can extend this line. [He extends the straight line p and finds one point of intersection. Jan nods.]



Teacher 11: OK. And now, is it possible to connect points K and L with a line so that it does not intersect straight line p or not?

Jan 12: No, it is not possible.

Dan 13: No, it is still possible, look, I can draw this line.

The teacher invites students to express a new conclusion. One that will not conflict with the existence of the intersection of two 'divergent' lines. Jan agrees with the conclusion, but does not express it himself. However, Dan does not notice any contradiction, so CoCo could not be induced.

Teacher 14: [pause] But you can extend the straight line again.

Dan 15: That's right, but I can draw it longer again.

Dan confirms that he does not resolve any CoCo. What the teacher perceives as a contradiction lies outside Dan's model. The curves in Dan's model arise dynamically and are not constant over time, as in the teacher's model. The core of the natural CoCo is therefore unattainable for Dan. (It is not entirely clear what the situation is with Jan. He agreed with the teacher and did not participate in the next situation. This would have to be determined by another task, which the teacher did not assign.)

<u>Case study No. 2: Oli and Niki, Grade 4; Teacher B</u>

Task: Arrange as many triangles as possible from 3, 4, and 5 wood sticks.

1) triangular inequality (the task for 4 sticks)
2) triangular inequality as a straight path and a path around
3) students will be able to use an argument of a longer path around for explaining the non-existence of 4 sticks-triangle

Teacher 16: OK, what about the triangles from 4 sticks?

Oli 17: Yes, I've got it.

Niki 18: Me as well. Show me it, we have the same one. I think there is only one possibility as in the case of 3 sticks. [Oli nods.]

Both students constructed the triangle that does not satisfy the triangular inequality. This situation means that there is a contradiction, as the teacher assumed, and it is possible to try to provoke CoCo.

Teacher 19: OK, how long are the sides of the triangle.

Oli 20: One, one and two sticks.

Teacher 22: Hm [pause] Mark the three vertices of your triangles on the paper. [pause, students point the dots] OK, and now write the lengths of the sides in the sticks.

Oli and Niki 23: [Students are writing the lengths: 1, 1, and 2.]

Teacher 24: OK, and now put your pencil to this point [teacher shows one of the vertices of the base] and go the shortest path to this point [teacher shows another one of the vertices of the base].

Oli and Niki 25: [Students show the path along the base.]

Teacher 26: How did you know that it is the shortest path?

Niki 27: Because we went straight ahead.

Teacher 28: OK, and how long was the path?

Oli 29: Two sticks.

Teacher 32: OK, and if I go this path? [teacher is showing the path via the third vertex]. What will it be like? Shorter, longer or the same?

Oli 33: Longer, of course.

Teacher 34: Do you agree, Niki?

Niki 35: Yes, sure!

CoCo could now be induced, but students still need to be reminded of their previous findings.

Teacher 36: OK, and what is the length of this path? [Teacher is showing the path via the third vertex again.]

Niki 37: One plus one [Niki shows the two sticks/numbers], it is two.

Oli 38: No, it couldn't be. This path is longer. [pause] It must be two plus something.

Oli realizes that he must reject his previous findings. Successfully resolves CoCo. His whole solution is actually hidden, and Oli declares the resulting shift.

Teacher 39: What do you think, Niki.

Niki 40: I don't know. It is longer than two sticks, but I don't know how much.

Teacher 41: I see. And what about the 4-stickstriangle? Is it alright?

Niki 42: Yes, it is.

Oli 43: No, it can't be like this.

Teacher 44: Yes, you are right. But how come you put it here?

Oli 45: It just looks like that, but it doesn't work that way.

Teacher 46: Niki, do you agree?

Niki 47: Yes, I do.

It seems that CoCo has also been caused by Niki and that she has successfully resolved it. Nevertheless, the teacher wants to verify this assumption.

[There is a part where students construct a 5-stickstriangle. Both students stated that there is only one such triangle and verified that triangle 3, 1, 1 cannot be constructed. Then the teacher asked about the 8-stickstriangle?]

Oli 63: I've got 3, 3, 2.

Niki 64: I've got 2, 2, 4.

Teacher 65: Can you check each other? [pause]

Oli 66: It can't be like this, it's the same as before. [Niki does not understand.] Here and here you are missing pieces. [Oli points to the sides of Niki's triangle.]

Niki 67: But you don't have it exactly.

Oli confirms that he accepted triangular inequality as a decisive argument for the existence of a triangle. Niki still makes decisions based on the visual impression. She does not understand Oli's argument.

Oli 68: Yeah, but this triangle works, yours doesn't. [pause]

Teacher 69: OK, and what about the triangle 1, 1, 4?

The teacher again tries to provoke a CoCo of the same nature, but with a stronger visual stimulus.

Niki and Oli 70: [Niki is trying to construct it. Oli shakes own head.]

Niki 71: It doesn't work. It's too short.

Teacher 72: Right. And what about 1, 2, 4?

Oli 73: [Oli shakes own head.]

Niki 74: It's still too short.

Teacher 75: OK, And what about the triangle 2, 2, 4?

Oli and Niki 76: [Oli shakes own head. Niki is trying to construct it again.]

Niki 77: It must be too short as well.

Teacher 78: Yes, well done!

Niki has come to the right conclusion, but it is not clear whether triangular inequality is an accepted concept for her. It seems that her visual and haptic experience will have to be repeatedly confronted as a sufficient argument for the existence of a triangle.

Case study No. 3: Kaja, Grade 6, Teacher A

Task: Where is the greater number of points on the circle *k* or on the circle *l*? (A figure of concentric circles has been attached.).

1) Larger and smaller lines/circles have got the same number of points. Students believe that a larger circle has more points than the smaller circle.
2) One-to-one correspondence between points on both circles.
3) Students adjust their belief that a larger circle has more points and will prove that the number of points is the same.

Teacher 8: OK, Yeah, so you're saying this circle has more points. I understand. [pause] Come try this with me now. Do you agree that a line that passes through the center has two intersections with the circle? [The teacher draws a circle and a line.]

Kaja 9: Yes, I do [Kaja points two intersections.] No, actually, this one more. [Kaja points to the center of the circle.]

Teacher 10: Hmm, but the center is not on the circle. It is an important point for the circle, but not its part.

Kaja 11: Of course, I know that. So only two.

Teacher 12: OK. And now imagine drawing lines running through the center of the two circles. Each will always have two and two intersections with each of the circles.

Kaja 13: That's right.

Teacher 14: You can draw a few to see. [Kaja draws three lines.] [pause] Do you still think that the smaller circle has fewer number of points?

Kaja 15: [pause] Yes, it must have.

The teacher has repeatedly (T12, T14) thought that it is already possible to induce CoCo because the teacher already records the contradiction. But Kaja doesn't realize it.

Teacher 16: Can you imagine that our lines have already passed through all the points of the smaller circle?

Kaja 17: Yes, they would be here everywhere.

Teacher 18. But would there still be any points left on the larger circle?

Kaja 19: [pause] Well, yes, because there are more of them.

Kaja's conviction is so steadfast that it does not allow for the presence of contradiction.

Teacher 20: And what if you drew another line there that passes through the center and some unoccupied point on the larger circle?

Kaja 21: It should be OK.

Teacher 22: And what about the smaller circle?

Kaja 23: [silence]

It can be assumed that Kaja admitted the presence of contradiction and that a CoCo was subsequently created in her, which she is trying to resolve.

Teacher 24: [pause] Will that line pass through even the smaller circle?

Kaja 25: Yes, [pause] it will [pause].

Teacher 26: So they should have an intersection, right?

Kaja 27: [pause] No, that line passes through, between two points.

Teacher 28: Wait, you said before that such a line always has an intersection with a circle.

Kaja 29: Yes, but not in this case. This is where it goes through.

Kaja resolved the CoCo. She rejected the correct and desirable statement from the teacher's (mathematics) point of view, so that she can retain the belief that a larger circle must have more points. Thus, the CoCo has been removed in an undesirable manner. It is, therefore, possible that Kaje was presented with this CoCo too soon, even when she was not ripe for its desired confrontation.

<u>Case study No. 4: Roman, pre-service teacher; Researcher B</u>

The following recording describes only a part of a larger conversation. In this part we focused on:

1) Two meanings of the word 'larger'. This situation is (maybe) specific for Czech language. First meaning corresponds with ordering of integers. The second meaning corresponds with ordering on natural numbers, based on measurement. This situation is based on a real pupil's question.
2) Transfer of knowledge about the ordering of positive integers (in the sense of measurement) to integers.
3) The teacher realizes that this is an incorrectly asked question.

Researcher 21: So, you said that a number 3 is greater than 2 and 2 is greater than 1.

Roman 22: Yes, that's true.

Researcher 23: What is the difference between numbers 3 and 2 and 2 and 1?

Roman 24: One, in both cases.

Researcher 25: OK. And how many times is a given number larger than another?

Roman 26: Twice and three times.

Researcher 27: Three times? Really? How do you mean?

Roman 28: The number 2 is twice as large as the number 1, and the number 3 is three times larger than the number 1.

Researcher 29: You are right. But I ask you about numbers 3 and 2.

Roman 30: Eee, sorry, it's my mistake. I think the number 3 is 1.5 times larger than the number 2.

…

Researcher 39: So, you said that a number -2 is greater than -3 and -1 is greater than -2.

Roman 40: Hmm, yes.

Researcher 41: What is the difference between numbers -3 and -2 and -2 and -1?

Roman 42: One.

Researcher 43: And now, how many times is the number -2 larger than the number -3?

Roman 44: He?

Researcher 45: And what about numbers 1 and -1? You will agree with me that the number 1 is larger than the number -1. And the difference is 2. So, how many times is the number 1 larger than the number -1?

Roman 46: [silence]

Now we have raised the CoCo in Roman's mind.

Roman 55: The number -1 is twice as large as the number -2, and the number -2 is 1.5 larger than the number [pause]. No, the number -2 is 0.5 larger than the number -3.

Researcher 56: Really? Is it a good concept that something may be half times larger than something else?

Roman 57: No, no,... Please, wait a minute. ... I don't know how many times is the number -1 larger than the number -2.

Researcher 58: OK. And what do you think, where is the problem? [pause] Do you believe, exists some information that is missing to you?

Roman 59: No. I think that I have all the needed information.

Researcher 60: Good. And what now?

Roman 61: I ... I don't know.

Roman is still in the CoCo and he doesn't deal with it. Roman evidently thinks about the problematic situation.

Roman 62: I think [pause] if 2 is twice larger than 1 and 4 is twice larger than 2. How many times is 1 larger than 0? [pause] How many times is something larger than 0? It is a strange question.

Researcher 63: Yes, you are right. And what now? What do you do with the last information? I think that the last remark is very important.

Roman 64: I'm sorry. I don't know. I see that there is something wrong, but now I don't know what it is.

Let's end this dialogue at this point. It is clear that the respondent was aware of the conflict that had occurred. Roman was mindful of the fact that the situation is wrong. Also, he believed that he had all the necessary knowledge to solve the problem but gave up. At this point, the following situation arose: There are two conflicting facts, but the student stopped dealing with them.

Conclusions

We presented four case studies in which teachers prepared didactic situations that led to contradictions. They expected that they would use them to evoke a CoCo, which the students would solve and thus develop a mathematical concept appropriately. In all four cases, however, the expected course did not occur. We described the reasons why the expected induction, or the expected solution of the CoCo, respectively, did not occur.

(1) The CoCo was not induced at all because the student does not register a contradiction. The presented contradiction lies in the model in which the teacher works but has no meaning in the model in which the student works. (Dan, case study No. 1) For the next shift, it is therefore necessary to influence the student's model, which can be helped by similar situations leading to contradictions.

(2) CoCo is induced, but a student is unable to reveal the concepts that evoke it. Therefore, they fail in its solution. (Roman, case study No. 4) The next didactic situation must first require a deeper understanding of the student's own ideas. Only then will the desired effect of exposure bring another contradiction.

(3) CoCo is induced, but students modify some of their concepts in an undesirable way. (Kaja, case study No. 3) Students were not prepared to resolve such a conflict and it is necessary to return to the reflected knowledge later in the future.

(4) CoCo is induced. The students are able to identify the concept that needs to be modified, but evaluate the statement only as an isolated situation. (Niki, case study No. 2) It can be assumed that when repeatedly exposed to an analogous contradiction or another contradiction with the same concept, the students gradually modify their concept accordingly.

(5) CoCo is induced and the students are able to identify the concept that needs to be modified and then perform their adjustment. (Oli, case study No. 2) However, even in this case, it is necessary to confront the existing understanding with new challenges for consolidating the acquired knowledge.[1]

If we compare our observations with Chan, Burtis, and Bereiter (1997) approach, we can state the following: The 'subassimilation' and 'direct assimilation' were not identified in the four case studies (With the exception of unclear induction or resolution of CoCo in the case of Jan (case study No. 1)). The 'surface-constructive' was identified in the case of Kaja (case study No. 3). She modified the present idea so that in this exceptional case that she did not have to review her own belief. The 'implicit knowledge building' was observed in the cases of Roman (case study No. 4), Niki, and Oli (case study No. 2), but each time with a different result. All of them identified CoCo and they perceived the problematic situation that needs to be explained. However, Roman was not able to do that. Niki did it, but only for a specific case. Oli was able to reconcile knowledge conflict successfully. In the end, we believe, we have not met the 'explicit knowledge building'. We believe that this requires higher metacognitive skills, which we do not anticipate in such old pupils.

Thus, we encountered various ways of confrontation with CoCo in the age group we studied. The described case studies indicate that the full breadth and complexity of the individual's cognitive structure needs to be considered here.

[1] Note that under existing resources, we are unable to decide how Jan (case study No. 1) resolved the cognitive conflict. Likewise, this may be the last of the solution described as the solution No. (2) or (4).

Based on these (and of course other) case studies, it is appropriate to plan didactic situations in order to provoke relevant CoCos, which will reflect all the mentioned possibilities of their students' confrontations. An appropriate connection would be to create concept cartoons using respondents' statements on the submitted tasks.

References

Adnyani, L.P.A.P. (2020). Applying Cognitive Conflict Strategy to Develop Mathematical Critical Thinking Ability and Character of Students. *Journal of Mathematics Education, 5*(1), 30-38.

Chan, C.K.K., Burtis, J. & Bereiter, C. (1997). Knowledge Building as a Mediator of Conflict in Conceptual Change. *Cognition and Instruction, 15*(1), 1-40.

Gazda, J., Liška, V., & Marek, B. (2019). *Kritické myšlení: Dovednost (nejen) pro 21. století* [Critical thinking: Skill (not only) for 21st century]. Prague: P3K.

Limón, M. (2001). On the cognitive conflict as an instructional strategy for conceptual change: a critical appraisal. *Learning and Instruction, 11*, 357-380.

Piaget, J. (2001). *Psychology of Intelligence*. London: Routledge.

Suh, J. & Seshaiyer, P. (2013). Mathematical Practices That Promote Twenty-First Century Skills. *Mathematics Teaching in the Middle School, 19*(3), 132–137.

Tall, D.O., & Schwarzenberger, R.L.E. (1978). Conflicts in the Learning of Real Numbers and Limits. *Mathematics Teaching, 82*, 44–49.