Zadání úloh

1. Řešte rovnici.

   (a) $\lfloor x + 1 \rfloor = 2$

   (b) $\lceil x + 1 \rceil = 2$

   (c) $5\lfloor x \rfloor - 4x = -11$

   (d) $4\lfloor x \rfloor - 5x = -11$

   (e) $2\lfloor x \rfloor - 7x = 7$

   (f) $2\lceil x \rceil - 7x = 7$

   (g) $2\lfloor x \rfloor - 5x = 9$

   (h) $\lfloor x^2 + 1 \rfloor = 2$

2. Určete počet řešení.

   (a) $x = \lceil x \rceil + \frac{x}{2024}$

   (b) $x = \lfloor x \rfloor + \frac{x}{2048}$

3. Načrtněte graf funkce.

   (a) $f(x) = \lfloor \frac{x}{2} \rfloor$

   (b) $f(x) = \lfloor x^2 \rfloor$

4. Dokažte.

   (a) $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$

   (b) Pro každé nezáporné celé číslo n platí $\left\lfloor \sqrt{2n(n+1)} \right\rfloor = \left\lfloor \sqrt{2} \cdot (n + \frac{1}{2}) \right\rfloor$.

   (c) Nechť $a, b \in \mathbb{R}$, $b > 0$, $n \in \mathbb{Z}$, $n > 0$. Potom

   $$\left\lfloor \frac{\left\lfloor \frac{a}{b} \right\rfloor}{n} \right\rfloor = \left\lfloor \frac{a}{nb} \right\rfloor.$$

5. Řešte rovnici.

   (a) $\log_3 \left(5 + 4 \log_2(x - 1) \right) = 2$

   (b) $\log_3(x + 2) + \log_3(x - 7) = 2 \log_3(x - 4)$

   (c) $5^{\frac{\log_3 x}{1 + \log_3 x}} = 0{,}2$

   (d) $\log_7(x - 7) + \log_7(x + 3) = 2 \log_7(x - 4)$

   (e) $\log_7(x - 3) + \log_7(x - 7) = 2 \log_7(x + 4)$

   (f) $\log_7(x - 5) + \log_7(x + 9) = 2 \log_7(x - 4)$